

# Fractional Order Sliding Mode Blood Glucose Control Using Functions Approximation Techniques

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**Abstract**—In this paper, based on function approximation techniques and sliding mode control strategy, a robust controller for blood glucose regulation is presented. Recently, there has been an increasing trend towards application of fractional order calculus in various fields. Healthcare of patients involved in diabetes mellitus is not exception and various researches have been carried out in this area. In order to design a robust controller, uncertainties should be estimated and compensated. Different estimators can be used for this purpose such as neural networks and fuzzy systems, due to their property of universal approximation. However, there are many tuning parameters in these estimators. Based on the orthogonal functions theorem, we can design other uncertainty estimators such as the Fourier series expansion and Legendre polynomials. Thus, in this paper, the Fourier series expansion is applied for uncertainty estimation in fractional order control of blood glucose. The efficiency of the proposed controller, i.e. robustness and high accuracy, in presence of physical disturbances like food intake and parametric uncertainties is verified via simulations.

**Keywords**-Fractional calculus; function approximation techniques; blood glucose regulation)

## I. INTRODUCTION(HEADING 1)

Diabetes is discussed as a serious condition in which the body's production and use of insulin are impaired, causing glucose concentration level to increase in the bloodstream. Insulin is a hormone generated by specific cells, called beta cells, in the pancreas. In order to transfer blood glucose into cells, insulin is required. Two types of diabetes have been recognized. In type I diabetes mellitus (T1DM), the b-cells in the pancreas that are responsible for producing insulin are destroyed by the immune system of the patients. Thus, the current solution for treatment is the delivery of exogenous insulin to maintain the glucose levels close to normal.

Based on continuous glucose monitoring (CGM) systems and insulin pumps technologies, a controller that automatically monitors and regulates the blood glucose level can be designed. In other words, it can play the role of an artificial pancreas system to replace the conventional treatment strategies in T1DM. In recent decades, various approaches have been presented in the literature for intelligent control of blood glucose. In this paper, the 3rd order minimal model of Bergman [1] is adopted.

Various approaches have been presented to design a feedback controller for blood glucose regulation, such as fuzzy logic control [2-5], recurrent neural networks [6], model predictive control (MPC) [7], high order sliding mode control [8], optimal control [9] and back-stepping sliding mode control [10]. Also, based on fractional order control, interesting approaches have been introduced in the field of blood glucose regulation [11-15].

Fractional calculus is a generalization of ordinary differentiation- integration operations to an arbitrary fractional order. This field of mathematics was viewed as an only theoretical topic with no practical applications for 300 years [16]. However, recently, it has been used in different fields of engineering and physics. Fractional-order controllers provide extra parameters for tuning. Hence, using this idea in modern nonlinear control techniques like SMC can be useful.

According to the universal approximation theorem [17], fuzzy estimators can approximate any real continuous nonlinear function. Also, this theorem has been developed for neural networks such as radial basis function networks (RBFN) [18]. Based on the orthogonal functions theorem [19], Fourier series can also approximate nonlinear functions. This theorem states that a set of basis functions that their mutual inner products are zero, can be used for approximation of nonlinear functions with small approximation error. In Fourier series, these basis functions are sinusoidal terms and in Legendre polynomials, these basis functions are some polynomials that satisfy the orthogonality condition. Moreover, this theorem states that the approximation error is bounded. Also, the coefficient of each basis function in the final representation of the desired nonlinear function can be calculated by some integrations. In other words, the space of nonlinear functions can be spanned by that set of orthogonal functions. In [20], the controller designed based on the Fourier series has been practically implemented. This uncertainty estimator is simpler and less computational. Using this estimator, some controllers have been presented [21-23] in the field of robust and adaptive control. Thus, in this paper, this estimator is adopted for robust regulation of blood glucose.

This paper is organized as follows. Section 2 introduces fractional calculus. Section 3, describes the glucose-insulin

model. Section 4 explains the Fourier series expansion. Section 5 develops the proposed controller. Section 6 illustrates simulation results. Finally, section 7 concludes the paper.

## II. FRACTIONAL CALCULUS

Let us first introduce Caputo definition and results needed here with respect to fractional calculus which will be used later.

**Definition 1.** The  $\alpha$  th-order Caputo fractional derivative of function  $f(t)$  is given by [24]

$${}_{t_0}^C D_t^\alpha f(t) = \frac{1}{\Gamma(n-\alpha)} \int_{t_0}^t \frac{f^{(n)}(\tau)}{(t-\tau)^{\alpha-n+1}} d\tau, \quad t > t_0 \quad (1)$$

where  $t_0$  and  $t$  are the bounds of the operation,  $n = \min\{k \in \mathbb{N}/k > \alpha > 0\}$ , and  $\Gamma(n)$  denotes the famous Gamma function, which is defined as

$$\Gamma(n) = \int_a^\infty t^{n-1} e^{-t} dt \quad (2)$$

**Definition 2.** Fractional integration of the order  $\alpha \geq 0$  of  $f \in L^1([0, T])$ , is defined as [25]

$${}_{t_0}^C I_t^\alpha f(t) = \frac{1}{\Gamma(\alpha)} \int_{t_0}^t (t-\tau)^{\alpha-1} f(\tau) d\tau \quad (3)$$

for  $t \in (0, T)$ .

**Definition 3.** The Caputo fractional derivative of order  $\alpha > 0$  of  $f \in C^n([0, T])$ , is defined as  $D^\alpha f(t) = I^{n-\alpha} D^n f(t)$ , where  $n = [\alpha]$  [25].

**Lemma 1:** The Caputo fractional derivative of a function in quadratic form is given by the following [26]

$$\frac{1}{2} {}_{t_0}^C D_t^\alpha (\mathbf{x}^T(t) \mathbf{P} \mathbf{x}(t)) \leq \mathbf{x}^T(t) \mathbf{P} {}_{t_0}^C D_t^\alpha \mathbf{x}(t), \quad \forall \alpha \in (0, 1], \forall t \geq t_0 \quad (4)$$

where  $\mathbf{x}(t) \in \mathfrak{R}^\beta$ ; and  $\mathbf{P} \in \mathfrak{R}^{\beta \times \beta} > 0$  is a constant matrix.

## III. GLUCOSE - INSULIN DYNAMIC

Many models for describing glucose-insulin process has been presented. Bergman's minimal model has been proposed in 1980 by Doctor Richard Bergman. The main advantage of the Bergman minimal model is its simplicity. Following is the Bergman Minimal Model (BeM) [27]

$$\begin{aligned} {}_{t_0}^C D_t^\alpha x_1(t) &= -p_1 [x_1(t) - G_b] - x_1(t)x_2(t) + D(t) \\ {}_{t_0}^C D_t^\alpha x_2(t) &= -p_2 x_2(t) + p_3 [x_3(t) - I_b] \\ {}_{t_0}^C D_t^\alpha x_3(t) &= -n [x_3(t) - I_b] + \gamma t [x_1(t) - G_b]^+ + u(t) \end{aligned} \quad (5)$$

in which  $x_1(t)$ ,  $x_2(t)$  and  $x_3(t)$  are plasma glucose concentration, the insulin influence on glucose concentration

reduction, and insulin concentration in plasma respectively,  $u(t) \in R$  is injected insulin rate in (mU/min),  $G_b$  is the basal pre-injection level of glucose (mg/dl),  $I_b$  is the basal pre-injection level of insulin ( $\mu$ U/ml),  $p_1$  the insulin independent rate constant of glucose uptake in muscles and liver (1/min),  $p_2$  the rate for decrease in tissue glucose uptake ability (1/min),  $p_3$  the insulin-dependent increase in glucose uptake ability in tissue per-unit of insulin concentration above the basal level ( $(\mu$ U/ml)/min). The term  $\gamma t [B_1(t) - G_b]^+$  represents the pancreatic insulin secretion after a meal in take at  $t = 0$ . As this work is focused on Insulin therapy which is usually administrated to Type I diabetes mellitus patients,  $\gamma$  is assumed to be zero to represent the true dynamic of this disease and  $p$  should also be considered zero. The parameter  $n$  is the first order decay rate for insulin in blood. This disturbance can be modeled by a decaying exponential function of the following form [28]:

$$D(t) = A \exp(-Bt) \quad B > 0 \quad (6)$$

The pump can be modeled as a first order linear system:

$${}_{t_0}^C D_t^\alpha u(t) = \frac{1}{a} (w(t) - u(t)) \quad (7)$$

where  $w(t)$  is insulin rate command in pump as input, and the parameter  $a$  is pump time constant.

## IV. THE FOURIER SERIES EXPANSION

According to [29], if the function  $F(t)$  defined on  $[t_1, t_2]$  satisfies the Dirichlet's conditions, then it can be expressed as

$$F(t) = a_0 + \sum_{k=1}^{\infty} a_k \cos(\omega_k t) + b_k \sin(\omega_k t) \quad (8)$$

where  $a_0$ ,  $a_k$  and  $b_k$  are the Fourier series coefficients,  $\omega_k = 2k\pi/T$  are the frequencies of sinusoidal functions and  $T$  is the fundamental period of  $F(t)$ . The truncation error is defined as

$$\varepsilon_m(t) = F(t) - F_m(t) \quad (9)$$

where  $F_m(t) = a_0 + \sum_{k=1}^m a_k \cos(\omega_k t) + b_k \sin(\omega_k t)$  is the Fourier series approximation.

$$a_0 = \frac{1}{T} \int_{t_1}^{t_2} g(t) dt \quad (10)$$

$$a_k = \frac{2}{T} \int_{t_1}^{t_2} f(t) \cos\left(\frac{2k\pi}{T} t\right) dt \quad (11)$$

$$b_k = \frac{2}{T} \int_{t_1}^{t_2} f(t) \sin\left(\frac{2k\pi}{T} t\right) dt \quad (12)$$

Note that  $F_m(t)$  can be written as

$$F_m(t) = \theta^T \xi(t) \quad (13)$$

in which

$$\theta = [a_0 \quad a_1 \quad b_1 \quad \dots \quad a_m \quad b_m]^T \quad (14)$$

and

$$\xi(t) = [1 \quad \cos(\omega_1 t) \quad \sin(\omega_1 t) \quad \dots \quad \cos(\omega_m t) \quad \sin(\omega_m t)]^T \quad (15)$$

It should be mentioned that in control systems, the function  $g(t)$  is not available and (10)-(12) cannot be applied for calculation of the Fourier series coefficients. In fact, these coefficients should be adjusted online using adaptation laws derived from the stability analysis.

## V. THE PROPOSED CONTROLLER

Taking the fractional derivative of (5) three times and using (7) results in

$${}^C D_t^{4\alpha} x_1(t) = f(t) + w(t) \quad (16)$$

where  $f(t)$  is a complicated function of  $x_1(t)$ ,  $x_2(t)$  and  $x_3(t)$  which is assumed unknown and should be estimated in the control law. In other words, based on (9), we have

$$f(t) = \theta^{*T} \xi(t) + \varepsilon(t) \quad (17)$$

$$\hat{f}(t) = \hat{\theta}^T \xi(t) \quad (18)$$

Assume that  $f^* = \theta^{*T} \xi(t)$  is the best approximation of  $f(t)$  using  $m$  frequencies. Since  $f(t)$  is uncertain,  $\theta^*$  is unknown and its estimation  $\hat{\theta}$  is used. The adaptation law tries to make  $\hat{\theta}$  converge to  $\theta^*$ . Define the sliding surface as

$$s = ({}^C D_t^\alpha e + \lambda)^3 \quad (19)$$

in which

$$e = x_1(t) - x_{1d}(t) \quad (20)$$

is the tracking error,  $x_{1d}(t)$  is the desired blood glucose level and  $\lambda$  is a design parameter. Also, (19) can be rewritten as

$$s = {}^C D_t^{3\alpha} e + 3\lambda {}^C D_t^{2\alpha} e + 3\lambda^2 {}^C D_t^\alpha e + \lambda^3 e \quad (21)$$

Taking the fractional derivative of (21) results in

$${}^C D_t^\alpha s = {}^C D_t^{4\alpha} e + 3\lambda {}^C D_t^{3\alpha} e + 3\lambda^2 {}^C D_t^{2\alpha} e + \lambda^3 {}^C D_t^\alpha e \quad (22)$$

Based on (20), we can write

$${}^C D_t^{4\alpha} e = {}^C D_t^{4\alpha} x_1(t) - {}^C D_t^{4\alpha} x_{1d}(t) \quad (23)$$

Substitution of  ${}^C D_t^{4\alpha} x_1(t)$  from (16) into (23) results in

$${}^C D_t^{4\alpha} e = f(t) + w(t) - {}^C D_t^{4\alpha} x_{1d}(t) \quad (24)$$

According to (24), (22) can be given by

$$\begin{aligned} {}^C D_t^\alpha s &= f(t) + w(t) - {}^C D_t^{4\alpha} x_{1d}(t) \\ &+ 3\lambda {}^C D_t^{3\alpha} e + 3\lambda^2 {}^C D_t^{2\alpha} e + \lambda^3 {}^C D_t^\alpha e \end{aligned} \quad (25)$$

Based on the sliding mode control strategy, the control law is obtained by  ${}^C D_t^\alpha s = 0$ . As a result, it follows from (25) that

$$\begin{aligned} w(t) &= {}^C D_t^{4\alpha} x_{1d}(t) - f(t) \\ &- 3\lambda {}^C D_t^{3\alpha} e - 3\lambda^2 {}^C D_t^{2\alpha} e - \lambda^3 {}^C D_t^\alpha e \end{aligned} \quad (26)$$

Since  $f(t)$  is unknown, it cannot be used in the control law and its estimation as given by (18) should be used. Thus, the control law is modified as

$$\begin{aligned} w(t) &= {}^C D_t^{4\alpha} x_{1d}(t) - \hat{f}(t) - u_r \\ &- 3\lambda {}^C D_t^{3\alpha} e - 3\lambda^2 {}^C D_t^{2\alpha} e - \lambda^3 {}^C D_t^\alpha e \end{aligned} \quad (27)$$

in which  $u_r$  has been considered for compensation of the truncation error of the Fourier series expansion  $\varepsilon(t)$ . Based on the orthogonal functions theorem that states the approximation error is bounded, we can assume that

$$|\varepsilon(t)| \leq \rho \quad (28)$$

in which  $\rho$  is a known positive constant. Using Lyapunov stability theorem, the adaptation law for  $\hat{\theta}$  and the robust control term  $u_r$  can be obtained as

$${}^C D_t^\alpha \hat{\theta} = \beta s \xi(t) \quad (29)$$

$$u_r = \rho \text{sign}(s) \quad (30)$$

in which Based on Barbalat's lemma [30], the asymptotic convergence of the tracking error can be guaranteed.

Table 1 The model parameters

Bergman minimal model	
$P_1(\text{min})^{-1}$	0
$P_2(\text{min})^{-1}$	0.0123
$P_3(\text{min})^{-1}$	$8.2 \times 10^{-8}$
$n(\text{min}^{-1})$	0.2659
$I_b$	7
$G_b$	70
$B_1(0)$	200

$B_3(0)$	50
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**Remark:** To attenuate the possible problem of chattering phenomenon caused by the sign function, the proportional-integrator structure proposed in [31-33] or the modification proposed in [34] can be used.

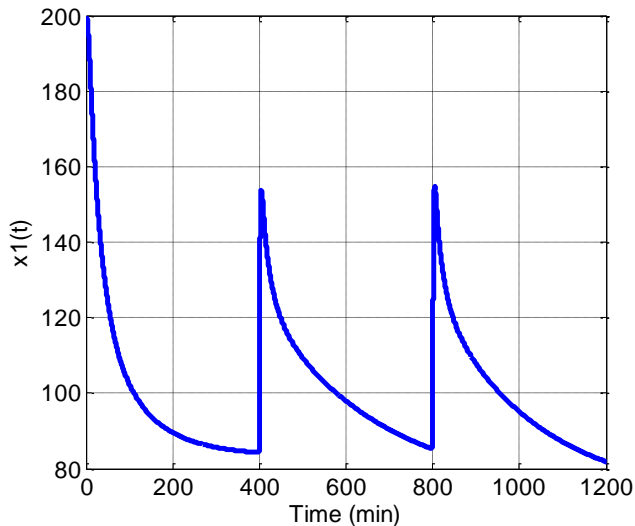


Fig.1 Glucose concentration

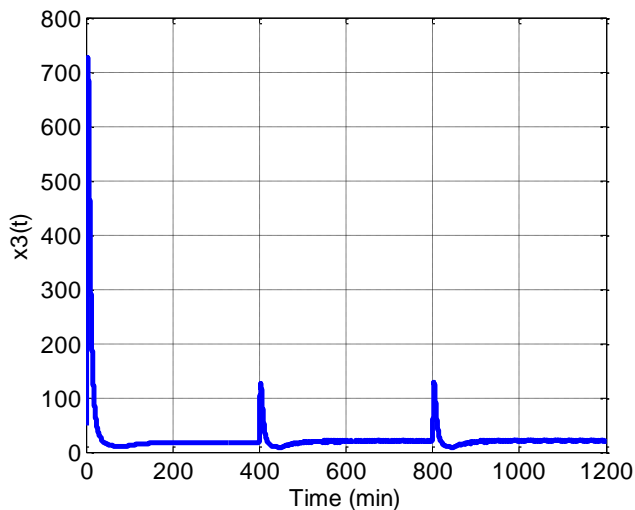


Fig. 2 Plasma insulin concentration

## VI. SIMULATION RESULTS

Consider the model described in [35]. Its parameters are given in Table 1. The parameter of the controller have been set to  $\lambda = 0.015$ ,  $\rho = 1$ ,  $\beta = 0.0001$ ,  $\alpha = 0.86$ . The blood glucose level is presented in Fig. 1. As shown in this figure, the controller can reduce the blood glucose concentration from the initial value of 200 (mg/dl) to the approximate value of 80 (mg/dl) which is our desired level. In comparison with controller designed in [35], the proposed controller is superior.

The reason is that when the external disturbance  $D(t) = 80\exp(-0.5t)$  affects the control system at  $t = 400, 800(\text{min})$ , the increase in the glucose level is less than that of the controller designed in [35].

The Plasma insulin concentration in (mU/L) is illustrated in Fig.2. As shown in this figure, the proposed controller outperforms the controller designed in [35]. The initial increase of Plasma insulin concentration for the proposed

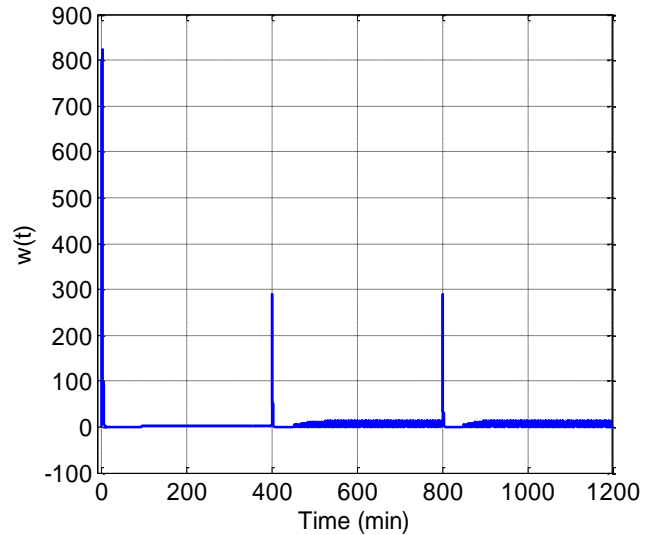


Fig. 3 Control low (insulin injection with pump)

controller is much less than that of the controller designed in [35]. The control signal  $w(t)$  which is the insulin injection with pump has been presented in Fig.3.

## CONCLUSION

In this paper, a fractional order controller for blood glucose regulation in type I diabetes patients has been presented. Uncertainties have been estimated and compensated using the Fourier series expansion which is less computational in comparison with other uncertainty estimators. The sliding mode control strategy has been adopted to make the controller robust against external disturbances. Simulation results verify the satisfactory performance of the proposed controller in comparison with a previous related work.

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